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SOLUTIONS OF GENERALITIES FOR SPECIAL TYPE OF WAVE CONTAINING TWO TIME AXES IN V4.

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ABSTRACT:
By Considering four dimensional plane symmetric line element containing the two times axes t_1 and t_2 as under:

$$ds^2 = -Ady^2 - \phi_2^2 Bdz^2 + \phi_3^2 2Bdt_1^2 + 2Bdt_2^2$$

The solutions of propagation equation of the generalities $R_{ij} = 0$ in empty region of space-times for $\left[z - \sqrt{t_1^2 + t_2^2}\right]$—type plane wave is found as

$$P = \frac{\bar{\bar{m}}}{2m} - \frac{\bar{m}^2}{4m^2} - \frac{\bar{m}\bar{B}}{2mB} + \frac{\bar{\bar{B}}}{2B} - \frac{3\bar{B}^2}{4B^2} = 0$$

Equivalent solutions are obtained by employing the concept of curvature tensor and Ricci tensor for above mentioned plane wave.

Keywords: - curvature tensor, Ricci tensor, plane wave solution, general theory of relativity, field equations, four dimensional space-times, generalities.

INTRODUCTION :

In the paper refer it to [1], Kadhao and Thengane, have obtained the plane wave solutions g_{ij} of field equations $R_{ij}= 0$ in four dimensional space-times V4 having two times axes for general theory of relativity as

$$\bar{\bar{w}}\rho_{\alpha\beta} + \bar{w}\sigma_{\alpha\beta} = \bar{\bar{\phi}}_2\rho_{\alpha\beta} + \bar{\phi}_2\sigma_{\alpha\beta} = \bar{\bar{\phi}}_3\rho_{\alpha\beta} + \bar{\phi}_3\sigma_{\alpha\beta} = 0 \quad (1)$$

where $\phi_2 = \frac{Z_{,2}}{Z_{,4}}, \quad \phi_3 = \frac{Z_{,3}}{Z_{,4}},$ (2)

$$w = \phi_\alpha x^\alpha = \phi_2 z + \phi_3 t_1 + t_2, \quad (3)$$

$$\sigma_{\alpha\beta} = -\bar{\rho}_{\alpha\beta} + \frac{1}{4}\left[\phi_\alpha\phi_\beta L_1 - 2L_2(\phi_\beta\rho_\alpha + \phi_\alpha\rho_\beta) + 2\rho_{\alpha\beta}\right]. \quad (4)$$

$$\rho_{\alpha\beta} = -\phi_\alpha\phi_\beta L_2 + \frac{1}{2}(\phi_\alpha\rho_\beta + \phi_\beta\rho_\alpha). \quad (5)$$

Also, we have established the existence of $\left[z - \sqrt{t_1^2 + t_2^2}\right]$—type plane waves in V4 with reference to the paper [2] and the solutions (1) reduced to

$$\bar{L}_2 - \bar{\rho}_4 + \frac{\rho_4^2}{2} - L_2\rho_4 + \frac{L_1}{4} = 0 \quad (6)$$

With the proper choice of coordinate system

$$g^{ij} = 0, \quad g_{ij} = 0, \quad i \neq j, \text{for } [i,j = 1,2,3,4] \quad (7)$$

we have investigated the general line element in V4 as

$$ds^2 = -Ady^2 - \phi_2^2 Bdz^2 + \phi_3^2 2Bdt_1^2 + 2Bdt_2^2 \quad (8)$$

In the present paper, we have studied the plane wave solutions (6) in detail for $\left[z - \sqrt{t_1^2 + t_2^2}\right]$—type plane wave using the line element (8).

2. $\left[z - \sqrt{t_1^2 + t_2^2}\right]$—type plane wave in V4

For $\left[z - \sqrt{t_1^2 + t_2^2}\right]$—type plane wave, the line element (8) becomes

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$$ds^2 = -Ady^2 - \left(\frac{t_1^2+t_2^2}{t_2^2}\right)Bdz^2 + \left(\frac{t_1}{t_2}\right)^2 2Bdt_1^2 + 2Bdt_2^2 \quad (9)$$

where A and B are functions of $Z = \left[z - \sqrt{t_1^2+t_2^2}\right]$

with

$$\phi_2 = \frac{Z_{,2}}{Z_{,4}} = \frac{-\sqrt{t_1^2+t_2^2}}{t_2}, \quad \phi_3 = \frac{Z_{,3}}{Z_{,4}} = \frac{t_1}{t_2}. \quad (10)$$

Then we get

$$v^i = \phi_\alpha g^{\alpha i} = \left[0,\ \left(\frac{t_2}{\sqrt{t_1^2+t_2^2}}\right)\frac{1}{B},\ \left(\frac{t_2}{t_1}\right)\frac{1}{2B},\ \frac{1}{2B}\right], \quad (11)$$

$$\rho_i = \bar{g}_{ij}v^j = \left[0,\ \left(\frac{-\sqrt{t_1^2+t_2^2}}{t_2}\right)\frac{\bar{B}}{B},\ \left(\frac{t_1}{t_2}\right)\frac{\bar{B}}{B},\ \frac{\bar{B}}{B}\right], \quad (12)$$

$$L_2 = \frac{\bar{m}}{2m} + \frac{3\bar{B}}{2B}, \quad (13)$$

$$L_1 = \frac{\bar{m}^2}{m^2} + \frac{3\bar{B}^2}{B^2}. \quad (14)$$

The field equations (6) then yield

$$P = \frac{\bar{\bar{m}}}{2m} - \frac{\bar{m}^2}{4m^2} - \frac{\bar{m}\bar{B}}{2mB} + \frac{\bar{\bar{B}}}{2B} - \frac{3\bar{B}^2}{4B^2} = 0 \quad (15)$$

3. $\left[z - \sqrt{t_1^2+t_2^2}\right]$—type plane wave in V_4.

Non-vanishing components of Christoffel symbol from (9) are calculated as follows.

$$\Gamma_{12}^1 = \frac{\bar{A}}{2A}, \quad \Gamma_{13}^1 = \left(\frac{-\bar{A}}{2A}\right)\frac{t_1}{\sqrt{t_1^2+t_2^2}}, \quad \Gamma_{14}^1 = \left(\frac{-\bar{A}}{2A}\right)\frac{t_2}{\sqrt{t_1^2+t_2^2}}$$

$$\Gamma_{11}^2 = \left(\frac{-\bar{A}}{2B}\right)\frac{t_2^2}{t_1^2+t_2^2}, \quad \Gamma_{11}^3 = \left(\frac{-\bar{A}}{4B}\right)\frac{t_2^2}{t_1\sqrt{t_1^2+t_2^2}}, \quad \Gamma_{11}^4 = \left(\frac{-\bar{A}}{4B}\right)\frac{t_2}{\sqrt{t_1^2+t_2^2}}$$

$$\Gamma_{22}^2 = \frac{\bar{B}}{2B}, \quad \Gamma_{22}^3 = \left(\frac{-\bar{B}}{4B}\right)\frac{\sqrt{t_1^2+t_2^2}}{t_1}, \quad \Gamma_{22}^4 = \left(\frac{-\bar{B}}{4B}\right)\frac{\sqrt{t_1^2+t_2^2}}{t_2}$$

$$\Gamma_{33}^2 = \left(\frac{\bar{B}}{B}\right)\frac{t_1^2}{t_1^2+t_2^2}, \quad \Gamma_{33}^3 = \left(\frac{-\bar{B}}{2B}\right)\frac{t_1}{\sqrt{t_1^2+t_2^2}}, \quad \Gamma_{33}^4 = \left(\frac{\bar{B}}{2B}\right)\frac{t_1^2}{t_2\sqrt{t_1^2+t_2^2}}$$

$$\Gamma_{44}^2 = \left(\frac{\bar{B}}{B}\right)\frac{t_2^2}{t_1^2+t_2^2}, \quad \Gamma_{44}^3 = \left(\frac{\bar{B}}{2B}\right)\frac{t_2^2}{t_1\sqrt{t_1^2+t_2^2}}, \quad \Gamma_{44}^4 = \left(\frac{-\bar{B}}{2B}\right)\frac{t_2}{\sqrt{t_1^2+t_2^2}},$$

$$\Gamma_{23}^3 = \frac{\bar{B}}{2B}, \quad \Gamma_{23}^2 = \left(\frac{-\bar{B}}{2B}\right)\frac{t_1}{\sqrt{t_1^2+t_2^2}}, \quad \Gamma_{24}^2 = \left(\frac{-\bar{B}}{2B}\right)\frac{t_2}{\sqrt{t_1^2+t_2^2}},$$

$$\Gamma_{24}^4 = \frac{\bar{B}}{2B}, \quad \Gamma_{34}^3 = \left(\frac{-\bar{B}}{2B}\right)\frac{t_2}{\sqrt{t_1^2+t_2^2}}, \quad \Gamma_{34}^4 = \left(\frac{-\bar{B}}{2B}\right)\frac{t_1}{\sqrt{t_1^2+t_2^2}}. \quad (16)$$

Non-vanishing components of curvature tensor in V_4 are as under

$$R_{1212} = \left[\frac{\bar{\bar{A}}}{2} - \frac{\bar{A}^2}{4A} - \frac{\bar{A}\bar{B}}{2B}\right], \quad R_{1213} = \left(\frac{-t_1}{\sqrt{t_1^2+t_2^2}}\right)\left[\frac{\bar{\bar{A}}}{2} - \frac{\bar{A}^2}{4A} - \frac{\bar{A}\bar{B}}{2B}\right],$$

$$R_{1214} = \left(\frac{-t_2}{\sqrt{t_1^2+t_2^2}}\right)\left[\frac{\bar{\bar{A}}}{2} - \frac{\bar{A}^2}{4A} - \frac{\bar{A}\bar{B}}{2B}\right], \quad R_{1313} = \left(\frac{t_1^2}{t_1^2+t_2^2}\right)\left[\frac{\bar{\bar{A}}}{2} - \frac{\bar{A}^2}{4A} - \frac{\bar{A}\bar{B}}{2B}\right],$$

$$R_{1314} = \left(\frac{t_1t_2}{t_1^2+t_2^2}\right)\left[\frac{\bar{\bar{A}}}{2} - \frac{\bar{A}^2}{4A} - \frac{\bar{A}\bar{B}}{2B}\right], \quad R_{1414} = \left(\frac{t_2^2}{t_1^2+t_2^2}\right)\left[\frac{\bar{\bar{A}}}{2} - \frac{\bar{A}^2}{4A} - \frac{\bar{A}\bar{B}}{2B}\right],$$

$$R_{2323} = -\left(\frac{t_1}{t_2}\right)^2\left[\frac{\bar{\bar{B}}}{2} - \frac{3}{4}\frac{\bar{B}^2}{B}\right], \quad R_{2324} = \left(\frac{t_1}{t_2}\right)\left[\frac{\bar{\bar{B}}}{2} - \frac{3}{4}\frac{\bar{B}^2}{B}\right],$$

$$R_{2334} = \left(\frac{-2t_1^2}{t_2\sqrt{t_1^2+t_2^2}}\right)\left[\frac{\bar{\bar{B}}}{2} - \frac{3}{4}\frac{\bar{B}^2}{B}\right], \quad R_{2424} = -\left[\frac{\bar{\bar{B}}}{2} - \frac{3}{4}\frac{\bar{B}^2}{B}\right],$$

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$R_{2434} = \left(\frac{2t_1}{\sqrt{t_1^2+t_2^2}}\right)\left[\frac{\bar{\bar{B}}}{2} - \frac{3}{4}\frac{\bar{B}^2}{B}\right]$, $\quad R_{3434} = \left(\frac{-4t_1^2}{t_1^2+t_2^2}\right)\left[\frac{\bar{\bar{B}}}{2} - \frac{3}{4}\frac{\bar{B}^2}{B}\right]$, (17)

They are related as

$$u = R_{1212} = \left(\frac{-\sqrt{t_1^2+t_2^2}}{t_1}\right)R_{1213} = \left(\frac{-\sqrt{t_1^2+t_2^2}}{t_2}\right)R_{1214} = \left(\frac{t_1^2+t_2^2}{t_1^2}\right)R_{1313} = \left(\frac{t_1^2+t_2^2}{t_1t_2}\right)R_{1314} = \left(\frac{t_1^2+t_2^2}{t_2^2}\right)R_{1414} \quad (18)$$

and

$$v = -\left(\frac{t_2}{t_1}\right)^2 R_{2323} = \left(\frac{t_2}{t_1}\right)R_{2324} = \left(\frac{-t_2\sqrt{t_1^2+t_2^2}}{2t_1^2}\right)R_{2334} = -R_{2424} = \left(\frac{\sqrt{t_1^2+t_2^2}}{2t_1}\right)R_{2434} = -\left(\frac{t_1^2+t_2^2}{4t_1^2}\right)R_{3434}. \quad (19)$$

where $\quad u = \frac{\bar{\bar{A}}}{2} - \frac{\bar{A}^2}{4A} - \frac{\bar{A}\bar{B}}{2B}$, (20)

$v = \frac{\bar{\bar{B}}}{2} - \frac{3}{4}\frac{\bar{B}^2}{B}$. (21)

And non-vanishing components of Ricci tensor are calculated as under

$$R_{22} = \frac{\bar{\bar{A}}}{2A} - \frac{\bar{A}^2}{4A^2} - \frac{\bar{A}\bar{B}}{2AB} + \frac{\bar{\bar{B}}}{2B} - \frac{3\bar{B}^2}{4B^2},$$

$$R_{23} = \left(\frac{-t_1}{\sqrt{t_1^2+t_2^2}}\right)\left[\frac{\bar{\bar{A}}}{2A} - \frac{\bar{A}^2}{4A^2} - \frac{\bar{A}\bar{B}}{2AB} + \frac{\bar{\bar{B}}}{2B} - \frac{3\bar{B}^2}{4B^2}\right],$$

$$R_{24} = \frac{-t_2}{\sqrt{t_1^2+t_2^2}}\left[\frac{\bar{\bar{A}}}{2A} - \frac{\bar{A}^2}{4A^2} - \frac{\bar{A}\bar{B}}{2AB} + \frac{\bar{\bar{B}}}{2B} - \frac{3\bar{B}^2}{4B^2}\right],$$

$$R_{34} = \left(\frac{t_1t_2}{t_1^2+t_2^2}\right)\left[\frac{\bar{\bar{A}}}{2A} - \frac{\bar{A}^2}{4A^2} - \frac{\bar{A}\bar{B}}{2AB} + \frac{\bar{\bar{B}}}{2B} - \frac{3\bar{B}^2}{4B^2}\right],$$

$$R_{33} = \left(\frac{t_1^2}{t_1^2+t_2^2}\right)\left[\frac{\bar{\bar{A}}}{2A} - \frac{\bar{A}^2}{4A^2} - \frac{\bar{A}\bar{B}}{2AB} + \frac{\bar{\bar{B}}}{2B} - \frac{3\bar{B}^2}{4B^2}\right],$$

$$R_{44} = \left(\frac{t_2^2}{t_1^2+t_2^2}\right)\left[\frac{\bar{\bar{A}}}{2A} - \frac{\bar{A}^2}{4A^2} - \frac{\bar{A}\bar{B}}{2AB} + \frac{\bar{\bar{B}}}{2B} - \frac{3\bar{B}^2}{4B^2}\right],$$

These are related as

$$P = R_{22} = \left(\frac{-\sqrt{t_1^2+t_2^2}}{t_1}\right)R_{23} = \left(\frac{-\sqrt{t_1^2+t_2^2}}{t_2}\right)R_{24} = \left(\frac{t_1^2+t_2^2}{t_1t_2}\right)R_{34} = \left(\frac{t_1^2+t_2^2}{t_1^2}\right)R_{33} = \left(\frac{t_1^2+t_2^2}{t_2^2}\right)R_{44} \quad (22)$$

where $\quad P = \frac{\bar{\bar{A}}}{2A} - \frac{\bar{A}^2}{4A^2} - \frac{\bar{A}\bar{B}}{2AB} + \frac{\bar{\bar{B}}}{2B} - \frac{3\bar{B}^2}{4B^2}$. (23)

The field equations (6) becomes

$$\frac{\bar{\bar{A}}}{2A} - \frac{\bar{A}^2}{4A^2} - \frac{\bar{A}\bar{B}}{2AB} + \frac{\bar{\bar{B}}}{2B} - \frac{3\bar{B}^2}{4B^2} = 0 \quad (24)$$

which is equivalent to (15).

CONCLUSION:

The plane wave solutions of generalities $R_{ij} = 0$ in four dimensional space-times V_4 for $\left[z - \sqrt{t_1^2+t_2^2}\right]$-type plane wave can be obtained by using the concept of curvature tensor as well as without using the concept of curvature tensor and found that both results are equivalent to each other.

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